

Numerical simulation and a hybrid method of supersonic turbulent base heating

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Abstract: Detached eddy simulation (DES) model was used to better capture large vortex structure and unsteady flow feature near cone base. Good base heating results were achieved using proper grid and numerical method. The influence of inflow conditions (like Mach number, Reynolds number) and configuration (bluntness ratio of the cone) on base heating was discussed based on numerical computational results. Compared with base heat flux, base pressure can be predicted more accurately by numerical simulation and the time consuming was shorter. A hybrid method combining numerical simulation with experimental correlation was proposed to predict supersonic turbulent base heating. A review of base heating experiment and correlations for slender cone was presented, and a revised correlation was proposed based on the above simulation results. Compared with the experiment data, the maximum error was 24%, a good result for base heating prediction. This proves that the hybrid method using the revised correlation performs well in predicting supersonic turbulent base heating of slender sphere cone and can meet needs of engineering design.

Key words: base heating; detached eddy simulation; hybrid method; experimental correlation; slender sphere cone

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Base heating of vehicle is an old problem of thermal protection system (TPS) design. Since heat flux in base area is much lower than that of fuselage, flow nature of separated near wake is not well understood yet. Computation uncertainty of base heating prediction is still large and conservative method is always used to design the TPS, which may increase the weight of TPS and definitely reduce vehicle's performance. The supersonic base thermal environment is influenced by near wake flow features, like separated shear layer, recirculation, reattachment, expansion wave and the shock. To better understand base heating, a brief description of the near wake flow is given, shown as Fig. 1. As illustrated in the figure, the flow expands near the shoulder, and this leads to large temperate and pressure

gradients around base edge. The free shear layer interacts with the lip shock and finally develops into far wake flow. Inside the recirculation region, slow air flow formed from the detachment affects the character of base heating. Therefore, base heat flux and pressure is related to accurate prediction of flow features in recirculation region.

The early studies on supersonic turbulent base heating utilize ground and flight experiments to establish correlations which could be used to the vehicle's aerothermal design. Bulmer^[1-2] presented turbulent base heat flux data and base heating correlation from re-entry vehicle (RV) flight experiments, which comprised wide range of hypersonic and supersonic inflow conditions. Zakkay and Cresci^[3] presented heat

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transfer and pressure distribution on the cone base from tunnel experiments. Francis^[4] presented base heating data from two RV flight experiments with different angle of attack.

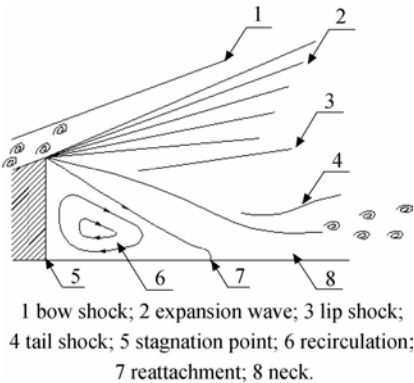


Fig. 1 Sketch of the near wake flow field

With the development of computational fluid dynamic (CFD) and increasing computational capabilities, there were many attempts in calculating base pressure using CFD approach. Lin, et al.^[5] analyzed near wake flow features and base pressure at different angles of attack using the Reynolds averaged Navier-Stokes (RANS) equations. Although near wake flow sketchy could be obtained by RANS, accurate base pressure and heat transfer were failed to predict. There were a few researches using large eddy simulation (LES) or direct numerical simulation (DNS) method to improve the calculating effect on base flow. Kawai, et al.^[6] used LES method to compute turbulent supersonic cylinder near wake flow. Sandberg, et al.^[7] used DNS method to compute cylinder base flow with Mach number 2.46. But these methods are stuck by the huge computing consumption. The detached eddy simulation (DES) is a hybrid model of RANS and LES. It was first presented by Spalart, et al.^[8], and then has gone through series of refinement. This method has the ability to obtain characteristic of detached flow by using similar LES method, and it is relatively lower costs of computation using RANS method near the wall. So it was used to solve the base flow problem. Forsythe, et al.^[9] utilized DES model to analyze cylinder base pressure. Barnhardt, et al.^[10] and

Chen, et al.^[11] used DES model on computing re-entry-F base flow. There were some researchers who studied cylinder base pressure showed DES model performed better in predicting base flow than RANS^[12-14]. But it is still worthy to re-search the effect when DES is used to calculate the base heat flux. The objective of this paper is to study supersonic turbulent base heating by using both DES and experimental correlations. The ability of DES and the hybrid method proposed in this paper on calculating heat flux in cone base were verified.

1 Numerical method

Flow field is obtained by solving three-dimensional Navier-Stokes equations, and the governing equation in integral form is given in the following,

$$\iiint_V \frac{\partial}{\partial t} \mathbf{Q} dV + \oint_{\partial V} \mathbf{F} dS + \oint_{\partial V} \mathbf{G} dS + \oint_{\partial V} \mathbf{H} dS = \oint_{\partial V} \mathbf{F}_v dS + \oint_{\partial V} \mathbf{G}_v dS + \oint_{\partial V} \mathbf{H}_v dS \tag{1}$$

in which

$$\begin{aligned} \mathbf{Q} &= \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e \end{bmatrix} & \mathbf{F} &= \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ u(\rho e + p) \end{bmatrix} \\ \mathbf{G} &= \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ v(\rho e + p) \end{bmatrix} & \mathbf{H} &= \begin{bmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + p \\ w(\rho e + p) \end{bmatrix} \\ \mathbf{F}_v &= \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ u\tau_{xx} + v\tau_{xy} + w\tau_{xz} + k \frac{\partial T}{\partial x} \end{bmatrix} \\ \mathbf{G}_v &= \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{yz} \\ u\tau_{xy} + v\tau_{yy} + w\tau_{yz} + k \frac{\partial T}{\partial y} \end{bmatrix} \end{aligned}$$

$$\mathbf{H}_v = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{yx} \\ \tau_{zz} \\ u\tau_{xx} + v\tau_{yx} + w\tau_{zz} + k\frac{\partial T}{\partial z} \end{bmatrix}$$

Where V and S are fluid element volume and element surface area respectively; u , v and w are velocity components; ρ is density; e is specific energy; k and T are thermal conductivity and the temperature respectively; τ represents viscous stress.

Detached eddy simulation used in the current study is the standard form^[8], which is based on Spalart Allmaras (SA) turbulent model^[15]. In both SA and LES model, eddy viscosity $\tilde{\nu}$ can be written as $\tilde{\nu} \propto \tilde{S}d^2$, where d means the distance from grid point to closest wall in SA model and grid spacing in LES, and $\tilde{S}$ is strain rate. In order to put both SA and LES model into one equation, a new function $\tilde{d}$ is proposed to replace d as following,

$$\tilde{d} = \min(d_{\text{rans}}, d_{\text{des}}) \tag{2}$$

$$d_{\text{rans}} = d \tag{3}$$

$$d_{\text{des}} = C_{\text{des}}\Delta \tag{4}$$

where Δ is the largest dimension of the local cell, $\Delta = \max(\Delta x, \Delta y, \Delta z)$; C_{des} is an adjustable parameter and set to 0.65 in this study.

In order to analyze the effect of different turbulent models, some results are given using $k-\omega$ shear-stress-transport (SST)^[16] and Delayed DES (DDES) model^[17].

2 Discussion of numerical results

2.1 Effects of grids and models on base flow and heat flux

Martellucci’s wind tunnel data was chosen to study the calculation effect of DES for supersonic base flow, which was obtained from Amold Engineering Development Center (AEDC) von Karman Gas Dynamics Facility (VKF) Tunnel B^[8]. The experiment geometry was a slender cone with base diameter 127 mm, the half cone angle 9° and the bluntness ratio 0.15. The inflow Mach number is 8 and the

Reynolds number is $1.8 \times 10^{-7} \text{m}^{-1}$. In the simulation, the outflow boundary uses supersonic extrapolation, and the solid wall boundary is treated as no slip and set isothermal condition (300 K). In the current study, second order time and spatial accuracy were used.

Three structured grids were used to examine the effect of grid number near wake region, as shown in Fig. 2. The numbers of grid points in the area behind the base are summarized in Table 1, where N_x , N_r and N_c represent the number of the grid points in the direction along the body, radius, and circle respectively. And N_t represents total number of the grid points.

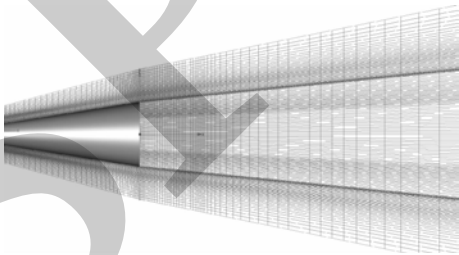


Fig. 2 Three-dimension computational grids

Table 1 Grid characteristics

Grid	N_x	N_r	N_c	$N_t/10^6$
Coarse	53	91	61	0.54
Medium	106	121	121	1.7
Refine	201	121	121	2.7

Fig. 3 shows flow properties (static pressure p , axial velocity u , static temperature T distributions) along the wake centerline, which is nondimensionalized by the inflow parameters (expressed by the subscript “inf” in the figure). The abscissa is given with the nondimensionalized distance, which is the ration of the distance from the base plane with the diameter of the base. Simulation results using DES model is compared to Martellucci’s wind tunnel data. The difference between the coarse grid results and the experiment results is larger than the medium and refine grid, which indicates that the medium grid is enough to simulation in this case. And the base flow sketchy and the parameters near

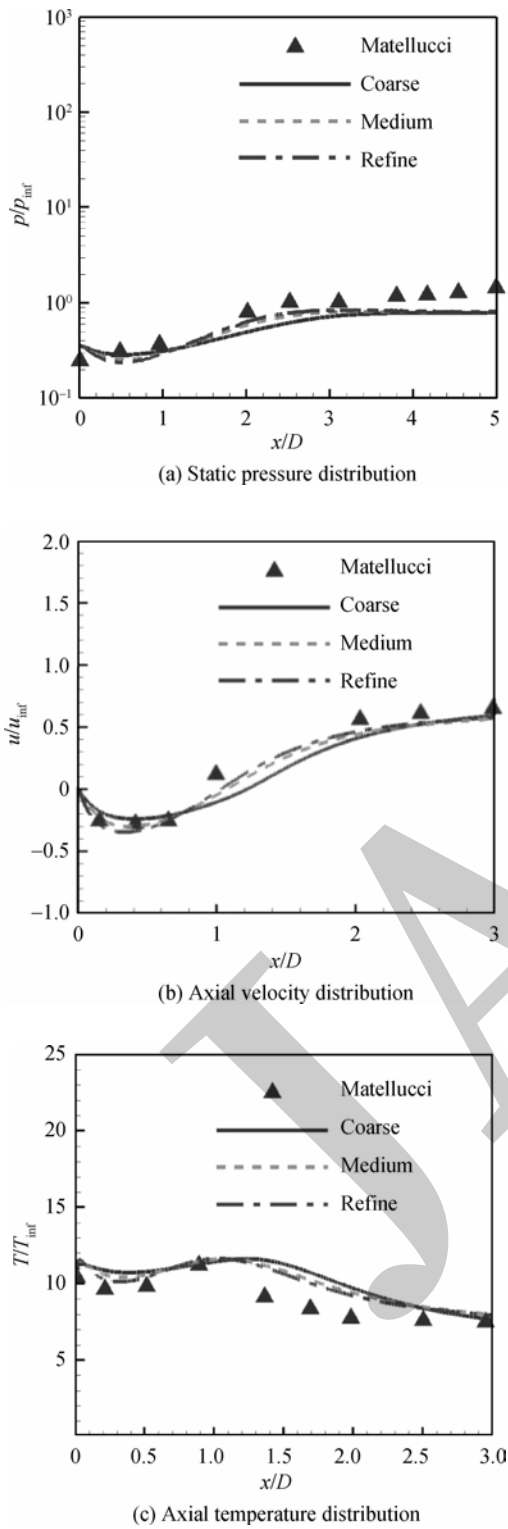


Fig. 3 Time-averaged flow properties along the wake centerline

the base can be calculated well by DES.

Fig. 4(a) shows static pressure (p) distributions along the radial direction of base. The abscissa is given with the nondimensionalized radius, which is the ration of the radius of the lo-

cation with the radius of the base. The results of three different grids approach the experiments when r/R_b is larger than 0.4. With the number of grids increasing, predicted base pressure is closer to the experiment. Fig. 4(b) shows heat flux distributions along the radial direction of base. The difference between the medium and refine grids is still evident compared with the pressure. It demonstrates that the heat flux is more sensitive to the grid than the pressure.

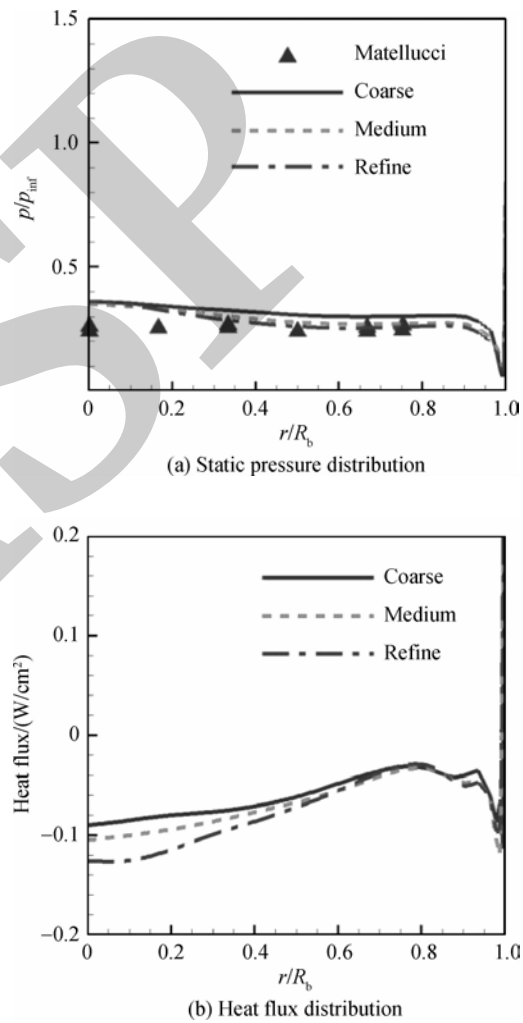


Fig. 4 Time-averaged flow properties along the base

Because heat flux was not measured in this experiment, another experiment case was calculated to evaluate the effect of DES on heat flux. The experiment was performed in a 1.2m hyper-sonic shock tunnel in Institute of Mechanics, Chinese Academy of Sciences^[18]. The half cone angle is 9° and the bluntness ratio is 0.1. The inflow Mach number is 8.5 and the Reynolds

number is $4.7 \times 10^7 \text{ m}^{-1}$. The experiment heat flux on the base center is $11\,702 \text{ W/m}^2$ in this case. The simulation results with different turbulent models and time steps are displayed in Table 2 and Table 3. It can be seen that the turbulent model and the time step have an obvious effect on the heat flux. The results of DES model are closer to the experiment value than RANS. With the decrease of the time step, the error between experiment and numerical result is reduced. The results indicate that the supersonic base heat flux can be calculated accurately by DES if the grid quantity is enough and the compute parameters are set up suitable.

Table 2 Numerical heat flux results using different turbulent models

Tubulent model	Heat flux on base center/(W/m ²)	Error between experiment and numerical result/%
RANS-SA	9 618.37	−17.8
RANS-SST	14 498.8	23.9
DES-SA	10 731.3	−8.3
DDES-SA	12 235.8	4.5

Table 3 Numerical heat flux results using different time steps

Time step	Heat flux on base center/(W/m ²)	Error between experiment and numerical result/%
1×10^{-5}	10 731.3	8.3
5×10^{-6}	11 066	5.5
1×10^{-6}	11 432.5	2.3

2.2 Effects of inflow condition on supersonic base heating

Using above numerical method, a series of cases with different inflow conditions (Mach number and Reynolds number) and configurations (bluntness ratio) based on the Martellucci’s tunnel experiment data^[5], was developed to analyze the effects of inflow condition on supersonic base heating. The flow conditions were shown in Table 4, in which Ma_{inf} , p_{inf} and Re_{inf} are inflow Mach number, pressure and Reynolds number;

R_n/R_b is bluntness ratio, and R_n is the nose radius of the slender sphere cone.

Table 4 Numerical Calculation Conditions

Case	Ma_{inf}	p_{inf}	$Re_{inf}/10^7$	R_n/R_b
M1	2	599.84	1.2	0.15
M2	4	599.84	1.2	0.15
M6	6	599.84	1.2	0.15
M8	8	599.84	1.2	0.15
M10	10	599.84	1.2	0.15
R7	8	335.91	7.0	0.15
R8	8	383.90	8.0	0.15
R9	8	431.88	9.0	0.15
R25	8	1204.48	2.5	0.15
B5	8	599.84	1.2	0.05
B20	8	599.84	1.2	0.20
B30	8	599.84	1.2	0.30

In Fig. 5, base pressure from simulated cases M2 to M10, in Table 4, compares to Bulmer’s four flight experiment data^[19]. “FLT2- $r/R_b=0.8$ ” in Fig. 5 represents pressure data measured in the location of $r/R_b=0.8$ on the base from the second flight and other symbols are similar. The flight experiment cones are 9° half-angle; 0.05 and 0.06 bluntness ratio; inflow Mach number throughout 0.5 to 15. Considering these experiment configurations are similar to Martellucci’s, those flight experiment

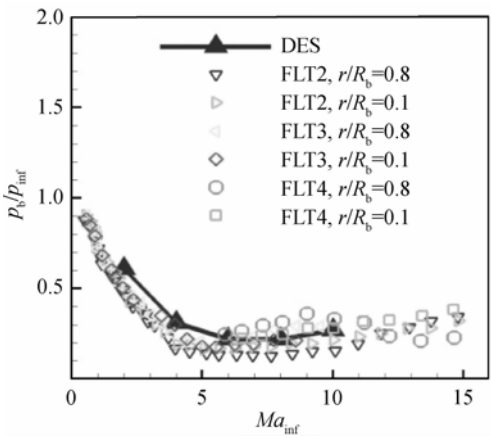


Fig. 5 Base pressures scaled with inflow pressure and Mach number

data were compared to our simulation results. From Fig. 5, our predicted base pressure matches well with the experiment data trends. Also, the figure exhibits a decreasing trend in p_b/p_{inf} as Ma_{inf} increasing in supersonic regime, and an increasing trend in hypersonic regime.

Fig. 6 shows nondimensional base pressure and Stanton number in the base center trends

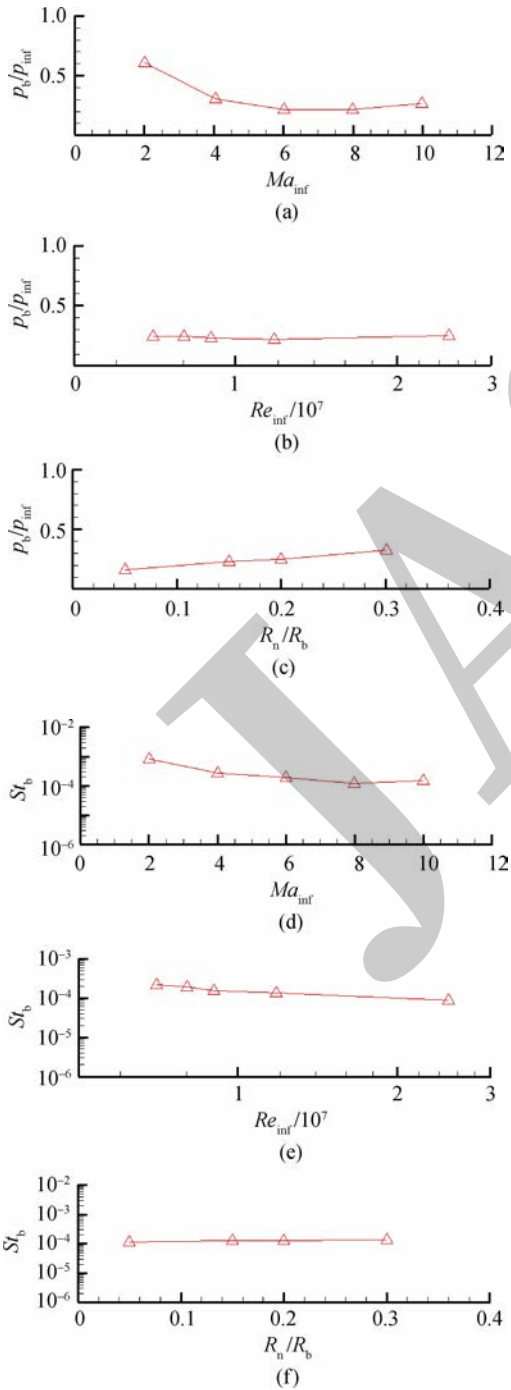


Fig. 6 Base center properties vs. flow conditions (Ma_{inf} , Re_{inf}) and configuration (R_n/R_b)

with different inflow Mach numbers, Reynolds numbers and bluntness ratios. In the center of the base, pressure decreases as Ma_{inf} and Re_{inf} increasing, and increases as bluntness ratio bluntness ratio R_n/R_b increasing. Heat flux decreases as Ma_{inf} and Re_{inf} increasing, and slightly increases as R_n/R_b increasing. However, the effect of Re_{inf} on base pressure and R_n/R_b on St are relatively faint.

3 A hybrid method to calculate supersonic base heat flux

3.1 Description of hybrid method

The accurate numerical results of supersonic base heating could be achieved by DES which need a lot computational experiences, not only because the base flow includes some complex phenomenon, but because the heat flux numerical results affected by many factors, such as grid quality and quantity, numerical scheme, turbulent model and so on. Compared with the heat flux, the influence factors of the pressure numerical results are less. And the convergence of the pressure is faster and the time consuming is much shorter than heat flux. So a hybrid method was proposed to predict the supersonic base heat flux, which combines the pressure numerical simulation results and the correlations of base heating.

Fig. 7 shows the process of the hybrid method. In the figure, $q_{r,n}$ and $q_{r,h}$ represent heat flux distribution calculated by numerical simulation and experimental correlation respectively. For a certain case, the first step is to solve the base flow field using numerical method (DES model), and base pressure can be obtained from this step. Because the objective of this step is the pressure, the requirement of the grid quantity and quality, numerical scheme are not very high. The computing time could be reduced. At this step, $q_{r,n}$ can be also achieved, but it is just a reference. After the accurate pressure was achieved, the heat flux $q_{r,h}$ can be obtained from heat correlation, because base heating correlates with pressure. Then the base

heating can be obtained by the hybrid method.

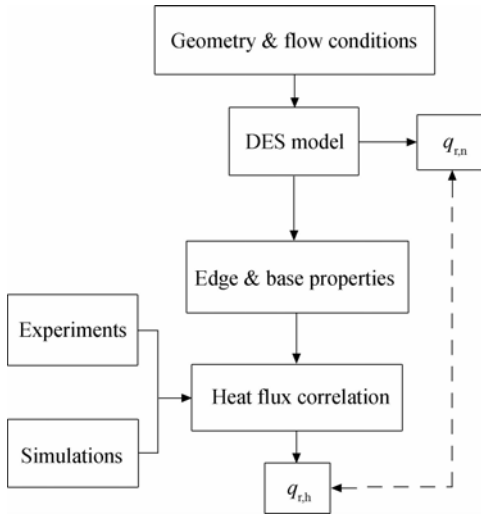


Fig. 7 Process of the hybrid method

3.2 Experimental correlation methods

For computing cone base heat flux, studies had been done to correlate base heat flux to other flow parameters (like base pressure, edge pressure and base temperature etc.). Bulmer's turbulent base heat-transfer correlation of slender cone is expressed as^[1]

$$\frac{Nu_{b,r}/Pr_b}{Nu_{e,sl}/Pr_e} = 35.5 \left(\frac{Re_{b,r}}{Re_{e,sl}} \right)^{2.2} \quad (5)$$

Where Nu is Nusselt number and Pr is Prandtl number, the subscript "b" means base; "r" means a certain location along radial direction in the base; "e" means the boundary edge; "sl" means wetted length of the cone.

Heat flux distribution along the base can be expressed as^[2]

$$\frac{q_r}{q_b} = (1 - r/R_b)^{1.2} \quad (6)$$

where q_r and q_b are heat flux in a location where the radius is r on the base and heat flux in base center respectively. This correlation derived from re-entry flight experiment data, and the experiment model is a 9° half angle sphere cone, bluntness ration (R_n/R_b) is 0.05, and the angle of attack was less than 0.5° .

Lamb converted Bulmer's correlation into a new equation using Stanton number (St), as showed in following equation^[20]

$$St_b = 8.3 \times 10^{-4} \left\{ \left[\frac{p_b}{p_{inf}} \left(\frac{p_{inf}}{p_e} \right)^2 \right]^{1.2} \right\}^{1.95} \quad (7)$$

$$q_b = p_{inf} u_{inf} St_b (h_{inf} - h_w) \quad (8)$$

Where St_b is Stanton number in base; p_b , p_e and p_{inf} are pressure of base, boundary edge and in-flow respectively. h_{inf} and h_w are inflow stagnation enthalpy and wall static enthalpy respectively.

3.3 Modification of base heating correlation

Based on the results of numerical calculation using DES model in section 2.3, a revised heat flux correlation on base center is expressed as

$$St_b = \{ 1.3 \times 10^{-3} [f(p_b, p_e)^{1.2} - 0.0004 \cdot (R_n/R_b - 0.05)] \} \cdot (1 - R_n/R_b)^{1.2} \quad (9)$$

$$f(p_b, p_e) = [(p_b/p_{inf}) \cdot (p_{inf}/p_e)^2]^{0.5} \quad (10)$$

This revised correlation is established from the results of slender cone which half angle is 9° , bluntness ratio ranges from 0.05 to 0.3, and inflow Mach number ranges from 2 to 10, Reynolds number ranges from 7×10^6 to 2.5×10^7 . Fig. 8(a) shows the base center Stanton number of DES and correlation results. The correlation results fit well with the simulation results. Fig. 8(b) shows heat flux distribution along the base achieved by DES simulation (case M10, R8, B5 in Table 4) and Bulmer's correlation^[2]. It can be seen that simulated heat flux distribution approaches Bulmer's correlation. Also, it suggests that heat flux decreases from base center to edge (x/R arranged from 0.2 to 0.8). So in the hybrid method, the correlation of heat flux distribution along the base continues using Bulmer's equation.

Two cases were computed to validate the hybrid method of heat flux on base center. Those cases are experimental studies of turbulent base heating of slender cones^[16]. Flow conditions and results are shown in Table 5. The hybrid method using the revised base heating correlation agrees well with Li's experimental data. The maximum error is 24%, which is reduced a lot compared with Lamb's correlation. It can be concluded that the hybrid simulation and correlation method are more appropriate to predict base heat flux in these cases.

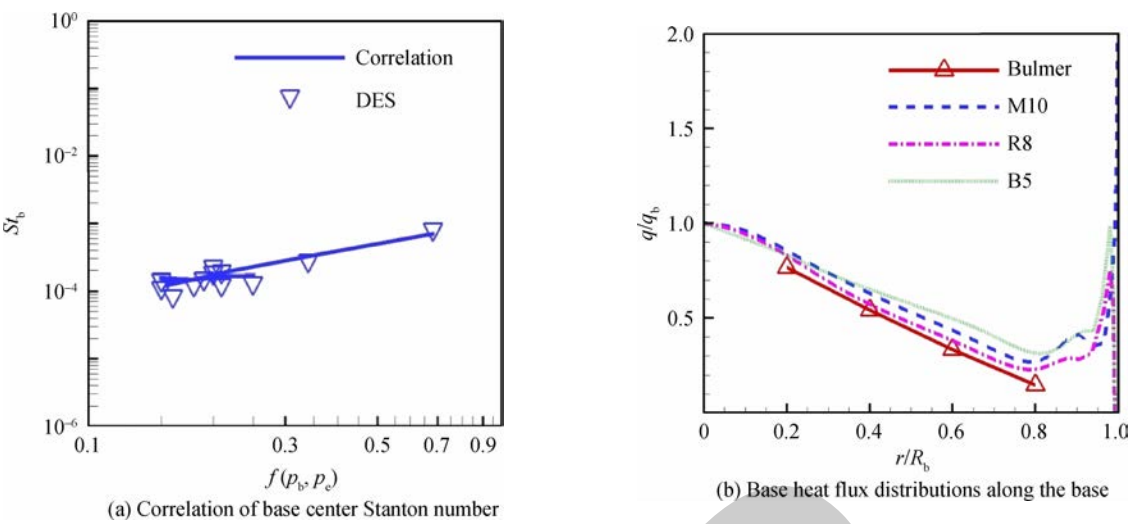


Fig. 8 Correlation of base heat flux

Table 5 Flow conditions and results of the turbulent base heating

No.	Ma_{inf}	p_{inf}/Pa	$h_{inf}/(J/kg)$	T_{inf}/K	Re_{inf}/m^{-1}	$\theta/(^{\circ})$	R_n/R_b	Heat flux/(W/m ²)		
								Experiment	Lamb's correlation	Hybrid method
1	8.5	2636.58	1003.2	64.6	4.7×10^7	9	0.1	11.8	3.0	14.6
2	8.5	2636.58	1003.2	64.6	4.7×10^7	9	0.2	14.3	5.1	17.8

4 Conclusions

DES is applied to calculate the hypersonic base heating in this paper. The base heat flux difference is below 10% if the grid quantity is enough and suitable. But it needs lots of computational experience and computer consuming. Effects of Mach number, Reynolds number, bluntness ratio on cone base heating were discussed. The results show that in the current computing conditions, base heat flux decreases as Mach number and Reynolds number increases, and the effect of bluntness ratio is relatively small. The results also show that base pressure can be calculated relatively accurately and is less affected by grid scale than base heat flux.

A hybrid method which combines numerical simulation with experimental correlation is proposed to predict supersonic turbulent base heating, in order to reduce computational consumption but not reduce computational accuracy. Using results of cases with different flow conditions, a correlation developed from Bulmer was

revised. The hybrid method using revised correlation was proved appropriate to predict supersonic turbulent base heating of slender cone. This revised correlation is based on the limited numerical results, and more works are needed to be done in the future to expand the application range.

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